$$x_{j+1} = c^{-1}a_{j,j+1}^{-1}, \tag{3.9}$$

$$y_{j+1} = a_{j+1,j}^{-1}, \tag{3.10}$$

$$x_k = -a_{kj}/a, \qquad y_k = 0, \qquad k = j+2, \cdots, n. \tag{3.11}$$

Now let $D(q)$ be I_{n-j} except for the $(1, 1)$ element which is q, let $e^t = (1, 0, \cdots, 0)$, and $m^t = (0, m_{j+2,j}, \cdots, m_{nj})$ where $m_{kj} = a_{kj}/a_{j+1,j}$. Then

$$I_{n-j} + axy^t = D(-a/b)(I_{n-j} - me^t),$$

$$I_{n-j} + bxy^t = (I_{n-j} + me^t)D(-b/a).$$

However $V_j(I_{n-j} - me^t)AV_j(I_{n-j} + me^t)$ represents the jth step of the reduction of A to tridiagonal form by elimination, see [1] and [3]. The matrices $D(-a/b)$ and $D(-b/a)$ represent the multiplication of row $j + 1$ and the division of column $j + 1$ by $-a/b$ and this leaves $S'_{j+1} = a'_{j+2,j+1}a'_{j+1,j+2}$ invariant for all permissible a, b. In general a, b are meant to be chosen so that $S'_{j+1} \neq 0$ and thus in this case they are nugatory and we may put $a = b = 1$.

This shows that the penultimate paragraph of p. 436 in [2] is not correct. For Hessenberg matrices the algorithm can break down.

New York University
Courant Institute of Mathematical Sciences
AEC Computing and Applied Mathematics Center
New York 3, New York

1. J. G. F. Francis & C. Strachey, "Reduction of a matrix to codiagonal form by eliminations," *Comput. J.*, v. 4, 1962, p. 168–176.
2. C. D. La Budde, "Reduction of an arbitrary real square matrix to tridiagonal form using similarity transformations," *Math. Comp.*, v. 17, 1963, p. 433–437.
3. J. H. Wilkinson, "Instability of the elimination method of reducing a matrix to tridiagonal form," *Comput. J.*, v. 5, 1962, p. 61–70.

A Note on Projective Planes of Order Nine

By E. T. Parker and R. B. Killgrove

Veblen and Wedderburn [1] in 1907 constructed three non-Desarguesian projective planes of order nine, two duals of one another and one self-dual plane. These planes, together with the Desarguesian, constitute the only projective planes of order nine discovered to date; it has, however, not been proved that no more exist.

Hall, Swift, and Killgrove [2], using the SWAC and card sorter, determined all permutation representations of planes of order nine, under the restriction that a set of nine parallel permutations forms the noncyclic group. No new planes were found; but, interestingly, more than one such coordinatization (i.e., nonisomorphic ternary rings with elementary abelian addition) was obtained for two known non-Desarguesian planes. Killgrove [3] carried out the same search for the cyclic group in place of the elementary abelian, obtaining no plane.

Among the latin squares in the coordinatizations of planes in [2], exactly five are nonisomorphic, exclusive of the group. This determination was facilitated con-

Received September 30, 1963.

siderably by counting numbers of lines consistent with every tabulated latin square —in the same sense that 2241 lines are consistent with the elementary abelian group (as described in [2]); a change of coordinate interpretation identifies these lines with transversals of the latin square. Then, knowing which classes of these latin squares were candidates for isomorphism, the mappings between latin squares were carried out by hand yielding five nonisomorphic examples.

0	3	6	2	7	4	1	8	5
1	4	7	0	8	5	2	6	3
2	5	8	1	6	3	0	7	4
3	6	0	7	4	2	8	5	1
4	7	1	8	5	0	6	3	2
5	8	2	6	3	1	7	4	0
6	0	3	4	2	7	5	1	8
7	1	4	5	0	8	3	2	6
8	2	5	3	1	6	4	0	7

0	2	1	6	8	7	3	5	4
1	0	2	8	7	6	5	4	3
2	1	0	7	6	8	4	3	5
3	8	7	0	5	4	6	2	1
4	7	6	5	0	3	2	1	8
5	6	8	4	3	0	1	7	2
6	5	4	3	2	1	0	8	7
7	4	3	2	1	5	8	0	6
8	3	5	1	4	2	7	6	0

0	3	6	2	7	4	1	8	5
1	8	3	5	6	2	0	7	4
2	6	4	0	8	5	7	1	3
3	0	8	7	4	1	2	5	6
4	2	5	8	1	0	6	3	7
5	1	7	6	2	3	8	4	0
6	4	0	1	3	7	5	2	8
7	5	2	4	0	8	3	6	1
8	7	1	3	5	6	4	0	2

0	5	7	8	3	2	4	1	6
1	7	3	6	2	4	5	0	8
2	6	5	7	4	0	3	8	1
3	8	1	4	0	6	2	7	5
4	1	8	2	6	3	0	5	7
5	0	6	3	8	1	7	4	2
6	2	4	1	7	5	8	3	0
7	3	0	5	1	8	6	2	4
8	4	2	0	5	7	1	6	3

0	6	3	4	1	7	8	5	2
1	5	6	0	4	8	2	3	7
2	3	7	1	5	6	0	4	8
3	7	2	5	6	1	4	8	0
4	8	0	3	7	2	5	6	1
5	2	8	6	3	0	1	7	4
6	1	5	8	0	4	7	2	3
7	4	1	2	8	5	3	0	6
8	0	4	7	2	3	6	1	5

THE FIVE NONISOMORPHIC LATIN SQUARES (NOT THE GROUP) IN [2]

For each of these five distinct latin squares, all extensions to planes were determined; no new planes were discovered. Depending on choice of coordinate interpretation, there are two versions of the result of this note: (1) *If L is a latin square in a ternary ring* (as in [2]) *coordinatization with elementary abelian addition of an order* 9 *plane, then any such coordinatization including L corresponds to a known plane.* (2) *If L is a latin square in a complete set of mutually orthogonal latin squares of order* 9 *including an elementary abelian group square, then any complete set of mutually orthogonal latin squares including L corresponds to a known plane.* (It is *not* asserted here that any completion of any latin square occurring in either sense in a representation of a known plane cannot yield a new type of plane; hence the specific mention of elementary abelian above.)

In determining completions to planes of each of the five latin squares, the procedures applied were as in [2]. Lines consistent with the square were generated by computer. The automorphism group of each latin square was worked out by studying cycle structures; in all five cases the groups were not really small. Lines were put into equivalence classes under the automorphism group. Finally, judicious use of preferences helped considerably in trimming down the size of the search. Again as in [2], it seemed most efficient to classify initially all admissible sets of lines through one point of the affine plane. One feature was easier than in [2]: whenever a plane was completed, it could be recognized (by the results of [2]) as a known plane if one of the latin squares displayed was the group, it being unnecessary to determine *which* known plane it was. In some eight cases exactly this happened; one case was a bit more stubborn, requiring projective completion and recoordinatization to yield a group square.

The computers used by the authors were respectively UNIVAC 1206 and SWAC. The effort was less a true collaboration than a division of labor arrangement.

The authors express thanks to Professors Gordon Pall and Esther Seiden for helpful suggestions relevant to this project.

UNIVAC Division of Sperry Rand Corporation
St. Paul, Minnesota
San Diego State College
San Diego, California

1. O. Veblen & J. H. M. Wedderburn, "Non-Desarguesian and non-Pascalian geometries," *Trans. Amer. Math. Soc.*, v. 8, 1907, p. 379–388.
2. Marshall Hall, Jr., J. Dean Swift & Raymond Killgrove, "On projective planes of order nine," *Math. Comp.*, v. 13, 1959, p. 233–246.
3. Raymond B. Killgrove, "A note on the nonexistence of certain projective planes of order nine," *Math. Comp.*, v. 14, 1960, p. 70–71.

The First Power of 2 With 8 Consecutive Zeros

By E. Karst and U. Karst

The existence proof for a string of k zeros within 2^n (in decimal notation) was recently given in [1]. Where those consecutive zeros occur the first time is another matter. We have written and run a fast program for the standard IBM 1620, discovering on January 1, 1964 the first string of 8 zeros, at $n = 14007$, after 1 hr. 18 min. There were no string of 9 zeros of 2^n up to $n = 50000$, which limit was reached after 13 hrs. 37 min. The first occurrences of 4, 5, 6, and 7 consecutive zeros, at $n = 377$, 1491, 1492, and 6801, respectively, as noted by Gruenberger [2], were checked and found correct. The string of 8 zeros in 2^{14007} starts at the 729th decimal digit position, reading from right to left.

Added in proof. On May 1, 1964, $n = 60000$ was reached. It takes now about one hour machine time to raise this upper bound by 2000.

Computation Center, University of Oklahoma
Norman, Oklahoma

1. Solution of E 1565, "A power of 2 with a string of n zeros," Amer. Math. Monthly, v. 71 1963, p. 1101–1102.
2. F. Gruenberger, "On strings of equal numbers within powers of two," Visiting lecture on the occasion of the University of Oklahoma Conference on Computer Science, Summer 1963.

Received January 6, 1964.